Cancellation of the sigma meson in thermal models

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The by now well-established scalar-isoscalar resonance $f_0(500)$ (the σ meson) seems potentially relevant in the evaluation of thermodynamic quantities of a hadronic gas, since its mass is low. However, we recall that its contribution to isospin-averaged observables is, to a surprising accuracy, canceled by the repulsion from the pion-pion scalar-isotensor channel. As a result, in practice one should not incorporate $f_0(500)$ in standard hadronic resonance-gas models for studies of isospin averaged quantities. In our analysis we use the formalism of the virial expansion, which allows one to calculate the thermal properties of an interacting hadron gas in terms of derivatives of the scattering phase shifts, hence in a model-independent way directly from experimentally accessible quantities. A similar cancellation mechanism occurs for the scalar kaonic interactions between the $I = 1/2$ channel (containing the alleged $K_0^*(800)$ or the κ meson) and the $I = 3/2$ channel.

I. INTRODUCTION

Nowadays, there is no doubt about the existence of the scalar-isoscalar resonance $f_0(500)$ (traditionally known as the σ meson). This very wide resonance is best described through its pole position, which according to the Particle Data Group (PDG) lies in a rather conservative range of $M_\sigma - i\Gamma_\sigma/2 = (400 - 550) - i(200\text{-}350)$ [1]. Such a low range for the mass was first reported in the 2012 version of the PDG tables [2], while previous editions quoted a very broad range 400-1200 MeV. Detailed studies of this resonance led to much smaller uncertainties: in Ref. [3] the pole at $\left(400 \pm 6^{+31}_{-13}\right) - i(278 \pm 6^{+34}_{-43})$ is obtained, while in Ref. [4–7] the value $(457^{+14}_{-13}) - i(279^{+11}_{-7})$ is quoted (for a compilation of all estimates and numerous references, see the mini-review 'Note on scalar mesons below 2 GeV' in Ref. [1]).

The question that we clarify in this paper is the role of the resonance $f_0(500)$ in thermal models for relativistic heavy-ion collisions (for reviews see, e.g., Refs. [8, 9] and references therein). Naively, one is tempted to include it as a usual but broad Breit-Wigner resonance, as was done for instance in Refs. [10, 11]. Notice that the pole mass of $f_0(500)$ is light (even lighter than the kaon), hence it might have a non-negligible effect on observables in more accurate studies. Indeed, in Ref. [11] the effect of $f_0(500)$ was found to increase the pion yield by 3.5%.

However, broad resonances should be treated with care. Moreover, there are also repulsive channels in the hadronic gas which must be taken into account on equal footing. The appropriate framework here is the virial expansion (see, e.g., Ref. [12]). There, one uses only stable particles (pions, kaons, nucleons, ...) as degrees of freedom, and their interactions are systematically incorporated in thermodynamics via coefficients of the virial expansion. The second term of this expansion, corresponding to the $2 \rightarrow 2$ reactions, is straightforward to include, as it involves the phase shifts accurately known from experiment. Within this approach, the attraction due to the pole of $f_0(500)$ is encoded in the pion-pion phase shift in the isoscalar-scalar ($I = J = 0$) channel. In principle, this attraction could generate non-negligible effects to thermodynamic properties. However, there is also repulsion from the isotensor-scalar ($I = 0, J = 2$) channel. It turns out that this repulsion, whose effect is by construction not included in standard thermal models, generates a negative contribution to the second virial coefficient. As a result, for isospin-averaged observables where the isotensor channel has degeneracy $(2I + 1) = 5$, one finds an almost exact cancellation of the positive contribution due to $f_0(500)$ from the isotensor channel. This feature has been reported already in Ref. [13], and more recently in Ref. [14], where similar conclusions were reached concerning the four-quark condensates and scalar susceptibilities of the hadron gas.

As a net result of the cancellation, the combined effect of $f_0(500)$ and the isotensor-scalar channel on all isospin-averaged properties of the hadron gas is negligible. This is the basic result discussed in our study.

A similar, albeit partial, cancellation occurs also in the pion-kaon S-wave interaction. The attraction in the channel $I = 1/2$ is canceled by a repulsion occurring in the $I = 3/2$ channel. In the attractive $I = 1/2$ channel many studies find a kaon-like resonance, called a $K_0^*(800)$ meson or κ [15, 16].

The framework of the virial expansion in the context of hot hadronic gas was used in numerous works, including the nature of the repulsive terms [13, 17–22], meson

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properties [23], thermal prediction from chiral perturbation theory [24], or the treatment of the $\Delta(1232)$ resonance [25–27].

The paper is organized as follows: in Sec. II we briefly present the formalism and explicitly show the cancellation between the isoscalar and the isotensor channels in the S-wave (and also the D-wave) channels. In Sec. III we illustrate the cancellation with the trace of the energy-momentum tensor as a function of T, as well as for the abundance of pions. In Sec. IV we describe the similar case of the kaonic resonance κ. Section V contains a discussion of two-particle correlation, where the cancellation mechanism does not occur and where $f_0(500)$ plays a relevant role [28]. Finally, in Sec. VI we present further discussion and conclusions.

II. INTERACTING PION GAS

A. The formalism

In modeling relativistic heavy-ion collisions, the study of hadron emission at freeze-out is successfully performed within thermal models (for reviews see, e.g., Refs. [8, 9] and references therein). In this framework, the free energy of the hadron gas at temperature T, i.e., $-k_B T \ln Z$, is represented as the sum of contributions of all (stable and resonance) hadrons, whence

$$\ln Z = \sum_k \ln Z_k^{\text{stable}} + \sum_k \ln Z_k^{\text{res}}, \qquad (1)$$

where k indicates the particle species. In practice, one uses the list of existing particles from the PDG [1],

In the limit where the decay widths of resonances are neglected, one has (for simplicity of notation we do not include chemical potentials)

$$\ln Z_k^{\text{stable,res}} = f_k V \int \frac{d^3 p}{(2\pi)^3} \ln \left[1 \pm e^{-E_p/T} \right]^{\pm 1}, \quad (2)$$

where f_k is the spin-isospin degeneracy factor, V is the volume, $\vec{p}$ is the momentum of the particle, the mass of the resonance is denoted as M_k, the energy is $E_p = \sqrt{\vec{p}^2 + M_k^2}$, and finally, the $\pm$ sign corresponds to fermions or bosons, respectively. Once $\ln Z$ is determined, all other thermodynamic quantities follow.

As a better approximation for the partition function, one can take into account the finite widths of resonances. The quantity $\ln Z_k$ is replaced by the integral over the distribution function $d_k(M)$:

$$\ln Z_k^{\text{res}} = f_k V \int_0^\infty d_k(M) \, dM \int \frac{d^3 p}{(2\pi)^3} \ln \left[1 - e^{-E_p/T} \right]^{-1}. \tag{3}$$

For narrow resonances one can approximate $d_k(M)$ with a (non-relativistic or relativistic) normalized Breit-Wigner function peaked at M_k. In the zero-width limit $d_k(M) = \delta(M - M_k)$ and Eq. (3) reduces to Eq. (2).

For the case where the resonance is broad, or when we need to include interactions between stable particles which do not lead to resonances at all (such as repulsion), we should use the virial expansion. Then the $2 \to 2$ reactions are incorporated according to the formalism of Dashen, Ma, Bernstein, and Rajaraman [29, 30]. The method, based on the virial expansion and valid for sufficiently low temperatures ($T \ll M$, where M is the invariant mass of the pair), uses as key ingredients the physical phase shifts which are well known for pion-pion scattering. Then

$$d_k(M) = \frac{d\delta_k(M)}{\pi dM}. \tag{4}$$

For simplicity of notation we focus on the case where the only degrees of freedom are pions. The partition function of the system up to second order in the virial expansion is given via the sum

$$\ln Z = \ln Z_\pi + f_{IJ} \int_0^\infty dM \frac{d\delta_{IJ}}{\pi dM} \int \frac{d^3 p}{(2\pi)^3} \ln \left[1 - e^{-E_p/T} \right]^{-1}, \tag{5}$$

where $\ln Z_\pi$ is the contribution from free pions evaluated according to Eq. (2), $f_{IJ} = (2I + 1)(2J + 1)$ is the spin-isospin degeneracy factor. In the case in which a series of narrow resonances labeled with k is present in the (I, J) channel, the derivative of the phase shift sharply peaks at the resonance positions according to

$$\frac{d\delta_{IJ}}{\pi dM} \simeq \sum_k \frac{\Gamma_{IJ,k}}{2\pi} \left[(M - M_{IJ,k})^2 + \frac{\Gamma_{IJ,k}^2}{4} \right]^{-1}, \quad (6)$$

thus one recovers the contribution of the (I, J) channel to the formula of Eq. (3) in the Breit-Wigner limit. Moreover, for $\Gamma_{IJ,k} \to 0$ one obtains $d\delta_{IJ}/(\pi dM) = \sum_k \delta(M - M_{IJ,k})$, in agreement with Eq. (2) [30].

Quantum-mechanically, the quantity $d\delta/(\pi dM)$ has a very simple interpretation: it is the difference of the density of two-particle scattering states of the interacting and free systems. For completeness, we present the derivation of this well-known fact in the Appendix.

B. The cancellation

In Fig. 1 we report the experimental $\pi\pi$ phase shifts as functions of the invariant mass up to 1 GeV and for different values of (I, J), as parametrized in Refs. [6, 7, 31]. From panel (a) it is clearly visible that the channel $(0, 0)$ is attractive (it is responsible for the emergence of the $f_0(500)$ pole), while the channel $(2, 0)$ is repulsive. The channel $(1, 1)$ is also attractive and corresponds to the prominent ρ meson. The isoscalar and isovector D-wave channels ($J = 2$) are reported in Fig. 1(b), showing that the interaction strength in these channels is negligible.

In Fig. 2 we present the derivatives of the phase shifts from Fig. 1, multiplied with f_{IJ}/π. We note from panel (a) that the resonance $f_0(500)$ does not lead to
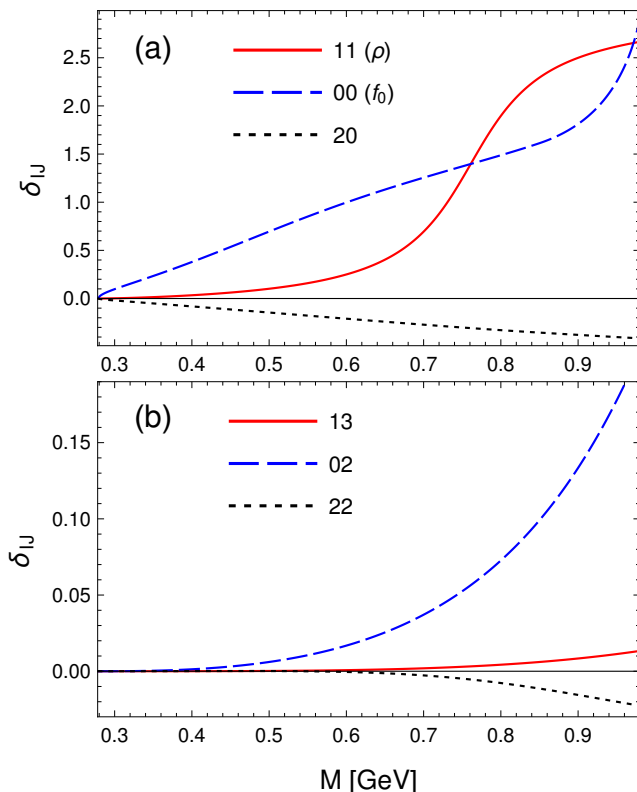
FIG. 1. (color online) The parametrization of the pion-pion scattering phase shifts plotted vs invariant mass for various isospin-spin (IJ) channels, taken from Ref. [6, 7, 31].
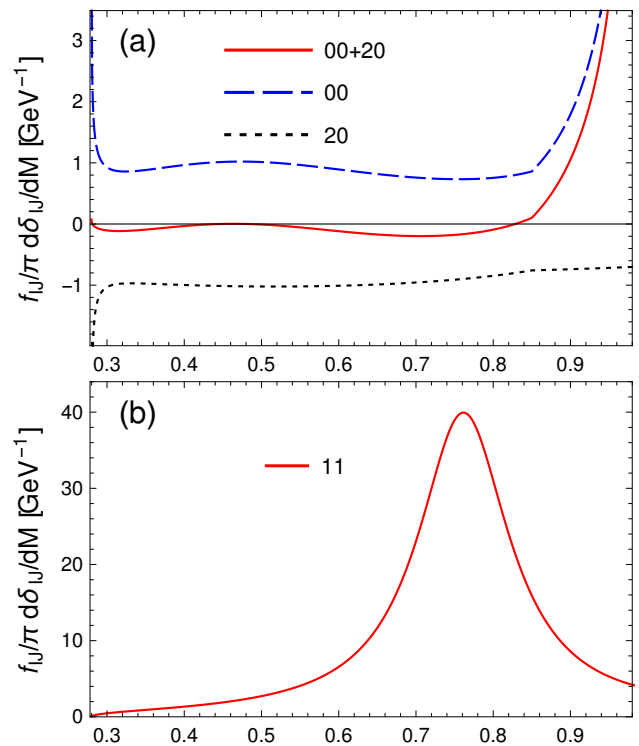

FIG. 2. (color online) The distributions $1/\pi f_{IJ}\, d\delta_{IJ}/dM$, corresponding to the phase shifts of Fig. 1, plotted as functions of the $\pi\pi$ invariant mass M. We note a nearly complete cancellation for the S-wave interactions between the $I = 0$ and $I = 2$ channels up to $M \sim 0.85$ GeV.

a pronounced peak, but rather to a smooth plateau with an integrable singularity close to the pion-pion threshold, which follows from kinematics. Importantly, the isotensor-scalar channel has a distribution, which up to $M \sim 0.85$ GeV is nearly a mirror reflection of the f_0 channel. Note that this is achieved with the multiplication of the isotensor channel by the isospin degeneracy factor $(2I + 1) = 5$, which occurs for isospin-averaged quantities (see Sec. V for other cases). As a result, the sum of the $(0,0)$ and $(2,0)$ channels nearly vanishes until $f_0(980)$ takes over above $M \sim 0.85$ GeV. We note that the negative phase shifts in certain channels may be attributed to repulsion due to the finite size of hadrons, see, e.g. [13, 17–22].

Recall that chiral perturbation theory (χPT) predicts cancellation between the S-wave isoscalar and isotensor channels (for a historical review see, e.g., Ref. [32]). The S-wave scattering lengths a_{I0}, related to the phase shifts near the threshold via $\delta_{I0} = q\, a_{I0} + o(q^2)$, with $q = \sqrt{M^2/4 - m_\pi^2}$, read explicitly at lowest order χPT [33]:

$$a_{00} = \frac{7}{32\pi}\frac{m_\pi}{f_\pi^2}, \quad a_{20} = -\frac{1}{16\pi}\frac{m_\pi}{f_\pi^2}. \tag{7}$$

We note the opposite signs, hence cancellation, while the relative strength between the isoscalar and isotensor channels is -3.5. Inclusion of chiral corrections [34] brings the ratio to the value -4.95 ± 0.16, while a global

analysis including dispersion relations made in Ref. [35] gives -4.79 ± 0.55. These values are compatible with -5 within the uncertainties. While χPT explains the cancellation near the threshold, its occurrence until as far as $M \sim 0.85$ GeV is a quite remarkable fact which goes beyond standard χPT.

In Fig. 2(b) we display the ρ-meson channel, which dominates the dynamics due to the fact that it is a pronounced resonance, and also carries a large spin-isospin degeneracy factor. In the remaining channels the interaction is very small. In addition, in the D-wave channel there is also a partial cancellation between the $(0,2)$ and $(2,2)$ channels at sufficiently low M.

III. CANCELLATION OF $f_0(500)$ IN THERMAL MODELS

As the cancellation between the attraction from $f_0(500)$ and the repulsion from the isotensor-scalar channel occurs at the level of the distribution functions $d_k(M)$, it will generically persist in all isospin-averaged observables. The purpose of the examples in this section is to show, how much error one would make by solely including the $f_0(500)$ and neglecting the isotensor repul-
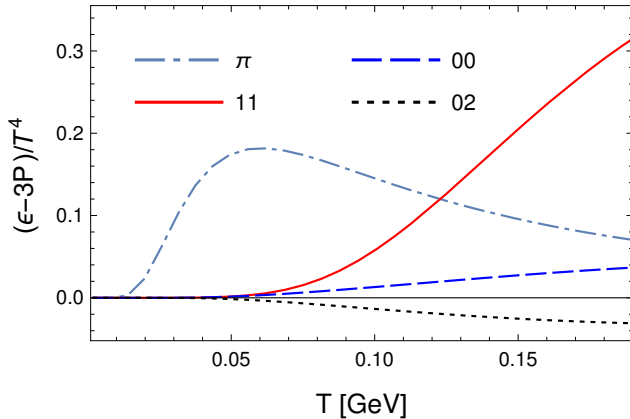
FIG. 3. (color online) Pion, ρ-meson, isoscalar-scalar, and isoscalar-tensor contributions to the volume density of the trace of the energy-momentum tensor divided by T^4, plotted as functions of T.
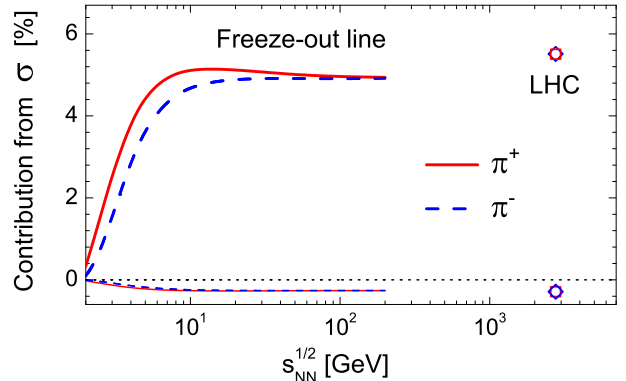

FIG. 4. (color online) Relative contribution to the pion yield from $f_0(500)$, evaluated with SHARE [10, 36] applied to A+A collisions at various collision energies. The thick lines correspond to the naive Breit-Wigner implementation of $f_0(500)$, while the thin lines indicate the correct inclusion of combined isoscalar-scalar and isoscalar-tensor channels. The points indicate the corresponding contributions at the LHC.

sion.

A. Trace of the energy-momentum tensor

In Fig. 3 we show the contribution to the trace of the energy-momentum tensor of the hadron gas from various channels: as expected, the contribution of a free pion gas is dominant at low T, while the contribution of the $I = J = 1$ channel (ρ meson) rises fast and overcomes that of free pions due to a large degeneracy factor. We note that the contribution from the attractive isoscalar-scalar channel (dashed line) is nearly canceled by the repulsive isotensor-scalar channel (dotted line).

B. Abundance of pions

The results of this subsection are of relevance to thermal modeling of particle production in ultra-relativistic heavy-ion collisions. In a simple version of this approach, hadrons (stable and resonances) are assumed to achieve thermal equilibrium, and later the resonances decay, feeding the observed yields of stable particles [37, 38]. The basic outcome are thus the abundances of various hadron species, or their ratios, which allow to determine the values of the thermal parameters at freeze-out. The contribution of light resonances is particularly important, therefore the treatment of $f_0(500)$ is relevant.

In our study we use the SHARE [10, 36] code to investigate the effect of $f_0(500)$ on pion abundances. The purpose of this study is to see, how much pions are generated from the naive approach, where $f_0(500)$ is improperly included as a wide Breit-Wigner pole with $M_\sigma = 484$ MeV and $\Gamma_\sigma/2 = 255$ MeV. In Fig. 4 we show the thermal model calculations carried out along the same freeze-out line as in [20], and for the temperature of $T = 156$ MeV and all chemical potentials equal to

zero at the LHC [37, 38] (the extra LHC points in Fig. 4). We find that with the naive implementation, the relative feeding of the $f_0(500)$ to the pion abundances (i.e., the ratio of the number of pions originating from the $f_0(500)$ to all pions) would be at a level of up to 5%, which is of a noticeable size and in agreement with the conclusions of Ref. [11].

However, in reality this contribution is not there due to the cancellation mechanism with the isotensor-scalar channel. The proper implementation of both the scalar-isoscalar and scalar-isotensor channels according to the phase-shift formula (4) for the density of states yields a very small negative contribution of the combined scalar-isoscalar and scalar-isotensor channels (at the level of -0.3%) to the pion yields, as indicated in Fig. 4 by the thin lines and the point below zero for the LHC.

We note that the proper implementation described above leads to lower pion yields, thus higher model hadron to pion multiplicity ratios, at a level of a few percent. Of particular interest here are the kaon to pion ratio, K^+/π^+, and the proton to pion ratio, p/π^+. For the former, the cancellation mechanism results in higher values of the thermal model results. This may help to describe the horn structure [39] at $\sqrt{s_{NN}} = 7.6$ GeV [40], where models fall somewhat below the data [11]. The latter is related to the LHC proton-to-pion puzzle [41], where the thermal model noticeably overpredicts p/π^+. With the cancellation of the $f_0(500)$ contribution, the proton-to-pion puzzle becomes even stronger, opening more space for possible novel interpretations [42].

IV. THE CASE OF $K_0^*(800)$

According to various works [15, 16], the attractive πK channel with $I = 1/2$ and $J = 0$ is capable of the generation of a pole corresponding to the putative resonance
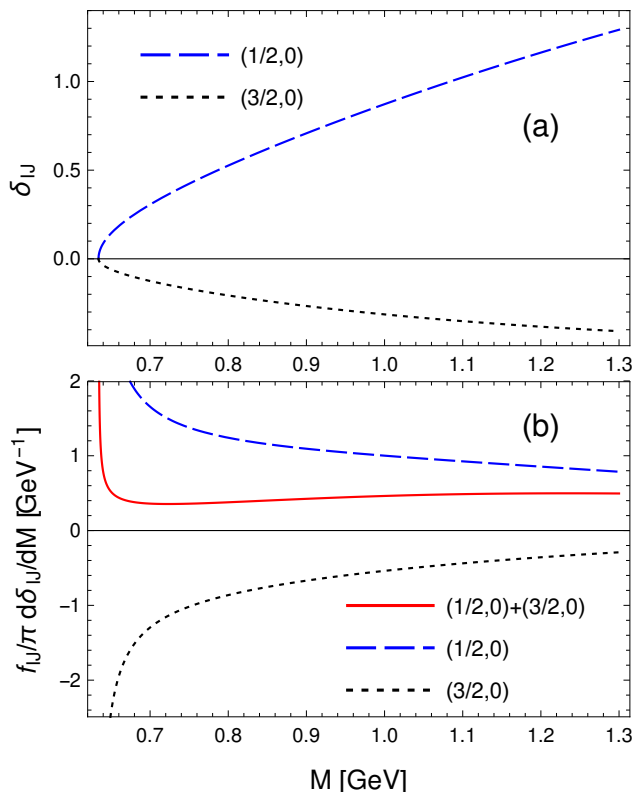
FIG. 5. (color online) The pion-kaon S-wave phase shifts (a) and their derivatives (b), multiplied with the isospin-spin degeneracy factors, plotted as functions of the invariant mass.

$K_0^*(800)$. This resonance has not been included in the summary table of the PDG, but it is naturally expected to exist as the isodoublet partner of the established resonances $f_0(500)$ or $f_0(980)$. This wide resonance has a predicted mass of ~ 680 MeV and a very large decay width of ~ 550 MeV, which makes its final assessment quite difficult (see also the discussion in 'Note on scalar mesons below 2 GeV' in Ref. [1]).

For the pion-kaon system, we use the effective-range fits to the phase shifts given in Ref. [43]. The results are plotted in Fig. 5, where the phase shifts and their derivatives (multiplied with the isospin-spin degeneracy factors) for the channels $(0, 1/2)$ and $(0, 3/2)$ are presented. Panel (b) also displays the sum (solid line), which exhibits the partial cancellation of the two contributions. Thus the conclusion here is similar, but less pronounced, compared to the case of the σ meson. The $I = 1/2$ channel is dominated by the vector channel $J = 1$, represented by the resonance $K^*(892)$.

V. CORRELATIONS

Up to now we have considered only isospin-averaged observables, where the isospin degeneracy factor of $(2I + 1)$ is present, leading to cancellation between the $(0, 0)$ and $(2, 0)$ channels. The cancellation does not occur for correlation variables. Suppose, as an example, that we investigate the correlated $\pi^+\pi^+$ pair production. Then, their source will be the isotensor channel only, and none will come from f_0.

An investigation of this type has been reported in Ref. [28], devoted to the analysis of the STAR Collaboration data [44] on resonance production. In particular, for the $\pi^+\pi^-$ pairs, the isospin Clebsch factors yield for the number of pairs

$$n_{\pi^+\pi^-}(M) = 3n_{\rho^0}(M) + \frac{2}{3}n_{f^0}(M) + \frac{1}{3}n_{(2,0)}(M), \quad (8)$$

where the factor of 3 in front of the ρ_0 contribution comes from the spin degeneracy. Since up to $M \sim 0.85$ GeV (cf. Fig. 2a) the ratio $n_{(2,0)}(M)/n_{f_0}(M) \simeq -1/5$, the relative yield of the isotensor channel compared to the isoscalar channel in Eq. (8) is only about 10%, and there is no cancellation. We thus clearly see the potential importance of the σ in studies of pion correlations [28].

VI. CONCLUSIONS

In this work we have presented, in the framework of the virial expansion, that the contribution of the resonance $f_0(500)$ (alias σ) to isospin-averaged observables (thermodynamic functions, pion yields) is nearly perfectly canceled by the repulsion from the isotensor-scalar channel. The cancellation occurs from a "conspiracy" of the isospin degeneracy factor, and the physical values for the derivative of the phase shifts with respect to the invariant mass. Thus, the working strategy in thermal models of hadron production in relativistic heavy-ion collisions, which incorporate stable hadrons and resonances, is to leave out $f_0(500)$ from the sum over resonances. Including it, would spuriously increase the pion yields at a level of a few percent. Alternatively, one may use physical phase shifts to do more accurate studies, but then, of course, all relevant isospin channels must be incorporated. The validity of the conclusion is for temperatures T smaller than the invariant mass of the hadronic pair. The lowest mass comes from the two-pion threshold, hence $T < 300$ MeV, which is a comfortable bound.

A similar cancellation occurs for $K_0^*(800)$ (alias κ) with quantum numbers $I = 1/2, J = 0$, whose contribution is compensated by the $I = 3/2, J = 0$ channel. On the other hand, in correlation studies of pion pair production, there is no cancellation mechanism.

ACKNOWLEDGMENTS

Appendix A: Derivation of the phase-shift formula

For completeness, in this Appendix we quote the quantum-mechanical derivation of the phase-shift formula of Eq. (3). The relative radial wave function of a pair of scattered particles with angular momentum l, interacting with a central potential, has the asymptotic behavior $\psi_l(r) \propto \sin[kr - l\pi/2 + \delta_l]$, where $k = |\vec{k}|$ is the length of the three-momentum, and δ_l is the phase shift. If we confine our system into a sphere of radius R, the condition $kR - l\pi/2 + \delta_l = n\pi$ with $n = 0, 1, 2, ...$ must be met, since $\psi_l(r)$ has to vanish at the boundary. Analogously, in a free system $kR - l\pi/2 = n_{\text{free}}\pi$. In the limit $R \to \infty$, upon subtraction,

$$\frac{\delta_l}{\pi} = n - n_{\text{free}}. \qquad (A1)$$

Differentiation with respect to M yields immediately the interpretation, that the distribution $d\delta_l/(\pi dM)$ is equal to the difference of the density of states in M of the interacting and free systems [12].

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